



WEST BENGAL STATE UNIVERSITY
B.Sc. Programme 5th Semester Examination, 2021-22

MTMGDSE01T-MATHEMATICS (DSE1)

Time Allotted: 2 Hours

Full Marks: 50

*The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.*

Answer Question No. 1 and any five from the rest

1. Answer any **five** questions from the following: 2×5 = 10
 - (a) Express $v = (x, y)$ as a linear combination of $v_1 = (1, 1)$ and $v_2 = (1, -1)$ in \mathbb{R}^2 .
 - (b) What 2 by 2 matrices represent the transformations that
 - (i) rotate every point by an angle θ about the origin.
 - (ii) reflect every point about the x -axis.
 - (c) What is the geometric object corresponding to the smallest subspace V_0 containing a nonzero vector $v = (r, s, t) \in \mathbb{R}^3$? Answer with reason.
 - (d) Write the matrix equation for the system of equations:

$$x + y = 3, -3y + 4z = 17, x - z = -8.$$
 - (e) Is there any straight line in the vector space R_2 which is a subspace of R_2 ?
 - (f) Find the inverse of the matrix $A = \begin{bmatrix} 5 & 3 \\ -2 & 2 \end{bmatrix}$.
 - (g) For what values of z the three vectors $(1, 1, 2)$, $(z, 1, 1)$ and $(1, 2, 1)$ are linearly independent?
 - (h) It is impossible for a system of linear equations to have exactly two solutions. Explain why.
 - (i) Prove that $\{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 = z^2\}$ is not a subspace of \mathbb{R}^3 .
2. (a) Examine if the set S is a subspace of \mathbf{R}_3 , $S = \{(x, y, z) \in \mathbf{R}_3 | x = 0, z = 0\}$. 4
 - (b) If $\alpha = (1, 2, 0)$, $\beta = (3, -1, 1)$, and $\gamma = (4, 1, 1)$, determine whether they are linearly dependent or not. 4

3. (a) If $A = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$, $B = \begin{bmatrix} p & q & r \\ s & t & u \end{bmatrix}$, $C = \begin{bmatrix} l & m \\ n & k \\ h & g \end{bmatrix}$, then establish that 4

$$(A+B)C = AC + BC.$$

(b) If $P = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$, and $Q = \begin{bmatrix} p_1 & p_2 & p_3 \\ q_1 & q_2 & q_3 \\ r_1 & r_2 & r_3 \end{bmatrix}$ then establish 4

(i) $(P+Q)^T = P^T + Q^T$ and (ii) $(P.Q)^T = Q^T.P^T$.

4. (a) Prove that two eigen vectors of a square matrix A over a field F corresponding to two distinct eigen values of A are linearly independent. 4

(b) Prove that the eigen values of a real symmetric matrix are all real. 4

5. (a) Prove that a matrix is non-singular if and only if it can be expressed as the product of a finite number of elementary matrices. 4

(b) Prove that if the rank of a real symmetric matrix be 1 then the diagonal elements of the matrix cannot be all zero. 4

6. (a) Diagonalize the matrix $A = \begin{bmatrix} 6 & 4 & -2 \\ 4 & 12 & -4 \\ -2 & -4 & 13 \end{bmatrix}$. 5

(b) Define a basis of a vector space. Do the vectors $(1, 1, 2)$, $(3, 5, 2)$ and $(1, 0, 0)$ form a basis of \mathbb{R}^3 ? Justify. 3

7. (a) Find the eigen vectors and eigenvalues of $\begin{bmatrix} 0 & 0 & 2 \\ 0 & 2 & 0 \\ 2 & 0 & 3 \end{bmatrix}$ 4

(b) If Q_θ represents the matrix for rotation (in x - y plane) through an angle θ about the origin, prove that $Q_\theta^2 = Q_{2\theta}$ and $Q_\theta Q_{-\theta} = I_2$ 4

8. (a) State Cayley-Hamilton's Theorem and verify it for the matrix $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & -1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$. 4

Hence find A^{-1} .

(b) What matrix has the effect of rotating every point through 90° and then projecting the result onto the x -axis? What matrix represents projection onto the x -axis followed by projection onto y -axis? 2+2

9. (a) Determine the rank of the matrix $\begin{bmatrix} 1 & 2 & 1 & 0 \\ 2 & 4 & 8 & 6 \\ 0 & 0 & 5 & 8 \\ 3 & 6 & 6 & 3 \end{bmatrix}$. 4

(b) If $A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 4 & -2 \\ -2 & 1 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$. Correct or justify: 4

(i) $(A-B)(A+B) = A^2 - B^2$

(ii) $(A-C)(A+C) = A^2 - C^2$

10.(a) Express $A = \begin{bmatrix} 2 & 5 & -3 \\ 7 & -1 & 1 \\ -1 & 3 & 4 \end{bmatrix}$ as a sum of a symmetric and skew symmetric matrix. 3

(b) If $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ and $C = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$ then verify that $AC = CA = 6I_3$ and 5

use this result to solve the system of equations

$$x - y = 3, 2x + 3y + 4z = 17, y + 2z = 7.$$

N.B. : Students have to complete submission of their Answer Scripts through E-mail / Whatsapp to their own respective colleges on the same day / date of examination within 1 hour after end of exam. University / College authorities will not be held responsible for wrong submission (at in proper address). Students are strongly advised not to submit multiple copies of the same answer script.

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