



WEST BENGAL STATE UNIVERSITY
B.Sc. Honours/Programme 4th Semester Examination, 2021

MTMHGEC04T/MTMGCOR04T-MATHEMATICS (GE4/DSC4)

Time Allotted: 2 Hours

Full Marks: 50

*The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.
All symbols are of usual significance.*

Answer Question No. 1 and any five from the rest

1. Answer any **five** questions from the following: 2×5 = 10
 - (a) Show that the set of cube roots of unity forms a group with respect to multiplication.
 - (b) In a group (G, \circ) prove that for all $a, b \in G$, $(a \circ b)^{-1} = b^{-1} \circ a^{-1}$.
 - (c) When a relation ρ defined on a nonempty set S is said to be an equivalence relation?
 - (d) Prove that in a commutative group G , the set $H = \{x \in G : x = x^{-1}\}$ forms a subgroup of G .
 - (e) Show that the group $(Z_4, +)$ is a cyclic group. Find its generators.
 - (f) Let R be a ring with 1. Show that the subset $T = \{n1 : n \in \mathbb{Z}\}$ is a subring of R .
 - (g) Show that the ring $(Z_5, +, -)$ is an integral domain.
 - (h) Determine whether the permutation f on the group S_5 is odd or even where
$$f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 4 & 5 & 3 & 1 & 2 \end{pmatrix}$$
 - (i) Define index of a subgroup H of a group G . If $G = S_3$ and $H = A_3$, then find the value of $[G : H]$.
2. (a) Let a relation R defined on the set \mathbb{Z} by “ $a R b$ if and only if $a - b$ is divisible by 5” for all $a, b \in \mathbb{Z}$. Show that R is an equivalence relation. 4
 - (b) Which of the following mathematical systems is / are group(s)? 2+2
 - (i) $(\mathbb{N}, *)$, where $a * b = a$ for all $a, b \in \mathbb{N}$.
 - (ii) $(\mathbb{Z}, *)$, where $a * b = a - b$ for all $a, b \in \mathbb{Z}$.
3. (a) Let the permutations f and g are the elements of S_5 where 2+2+1

$$f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 4 & 3 & 5 & 1 \end{pmatrix}, \quad g = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 5 & 4 & 1 \end{pmatrix}.$$
 Find fg , gf , f^{-1} .

- (b) Let $f: \mathbb{Z} \rightarrow \mathbb{Z}$ is defined by $f(n) = n^2$, $n \in \mathbb{Z}$ and $g: \mathbb{Z} \rightarrow \mathbb{Z}$ is defined by $g(n) = 2n$, $n \in \mathbb{Z}$. Find the composition of the functions $f \circ g$ and $g \circ f$. 2+1
4. (a) Verify the statement is true or false: In ring R if $(a+b)^2 = a^2 + 2ab + b^2$ for all $a, b \in R$, then R is a commutative ring. 3
- (b) (i) Show that the set $S = \left\{ \begin{bmatrix} x & 0 \\ 0 & x \end{bmatrix} \right\}$, $x \neq 0$ is a subgroup of the group of all 2×2 order non-singular real matrices. 2
- (ii) Let (G, \circ) be a commutative group and $H = \{a^2 : a \in G\}$, prove that H is sub-group of G . 3
5. (a) Prove that every subgroup of a cyclic group is cyclic. 4
- (b) Let G be a group of prime order. Then prove that G is cyclic. 4
6. (a) Find all right cosets of the subgroup $6\mathbb{Z}$ in the group $(\mathbb{Z}, +)$. 4
- (b) Let G be a group such that every cyclic subgroup of G is a normal subgroup of G . Prove that every subgroup of G is a normal subgroup of G . 4
7. (a) Let H be the set of all real matrices $\left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} : \det \begin{pmatrix} a & b \\ c & d \end{pmatrix} = 1 \right\}$. Prove that H is a subset of $GL(2, \mathbb{R})$. 4
- (b) Find all cyclic subgroups of the group (S, \cdot) , where $S = \{1, i, -1, -i\}$. 4
8. (a) Examine if the ring of matrices $\left\{ \begin{pmatrix} a & b \\ 2b & a \end{pmatrix} : a, b \in \mathbb{R} \right\}$ is a field. 4
- (b) Prove that a finite integral domain is a field. 4
9. (a) Show that the ring of matrices $\left\{ \begin{pmatrix} 2a & 0 \\ 0 & 2b \end{pmatrix} : a, b \in \mathbb{Z} \right\}$ contains divisors of zeros and does not contain the unity. 4
- (b) Prove that the ring $(\mathbb{Z}_n, +, \cdot)$ is an integral domain if and only if n is prime. 4
10. (a) Show that $T = \{[0], [5]\}$ is a subring of the ring \mathbb{Z}_{10} . 4
- (b) Let I and J be ideals of a ring R . Prove that $I + J$ is an ideal of R . 4

N.B. : Students have to complete submission of their Answer Scripts through E-mail / Whatsapp to their own respective colleges on the same day / date of examination within 1 hour after end of exam. University / College authorities will not be held responsible for wrong submission (at in proper address). Students are strongly advised not to submit multiple copies of the same answer script.

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